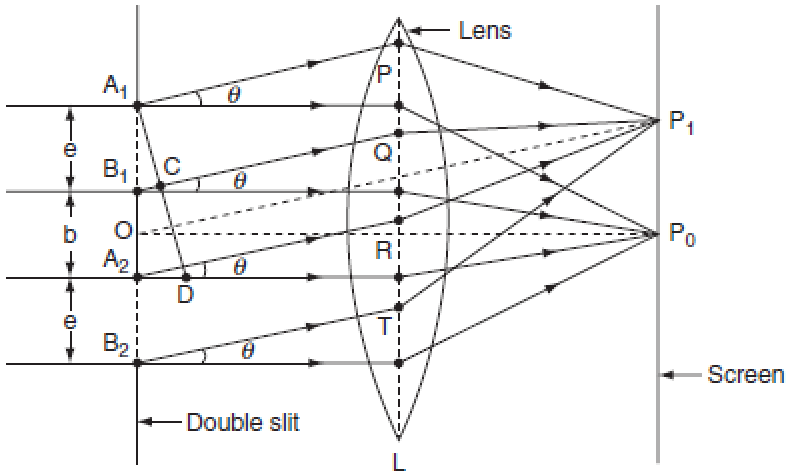
Diffraction due to double slit:

The double slits have been represented as A_1B_1 and A_2B_2 in Fig.1. The slits are narrow and rectangular in shape. The plane of the slits are perpendicular to plane of the paper. Let the width of both the slits be equal and it is 'e' and they are separated by opaque length'd'. A monochromatic plane wave front of wave length 'λ' is incident normally on both the slits.



Light is made incident on arrangement of double slit. The secondary wavelets travelling in the direction of OP_0 are brought to focus at P_0 on the screen SS' by using a converging lens L. P_0 corresponds to the position of the central bright maximum. The intensity distribution on the screen is the combined effect of interference of diffracted secondary waves from the slits.

The diffracted intensity on the screen is very large along the direction of incident beam [i.e along OP_0]. Hence it is maximum at P_0. This is known as principal maximum of zero order.

The intensity at point P_1 on the screen is obtained by applying the Fraunhofer diffraction theory at single slit and interference of diffracted waves from the two slits. The diffracted wave amplitude due to single slit at an angle θ with respect to incident beam is $A\frac{\sin\alpha}{\alpha}$, where 2α is the phase difference between the secondary wavelets arising at the end points of a slit. This phase difference can be estimated as follows: Draw a normal from A_1 to B_1Q. Now, B_1C is the path difference between the diffracted waves at an angle 'θ' at the slit A_1B_1.

From the triangle A_1B_1C

$\sin\theta = \frac{B_1C}{A_1B_1} = \frac{B_1C}{e}$ or $B_1C = e\sin\theta.$

The corresponding phase difference $(2\alpha) = \frac{2\pi}{\lambda} e\sin\theta$

$$or, \alpha = \frac{\pi e Sin\theta}{\lambda} \ldots\ldots\ldots\ldots..(1)$$

The diffracted wave amplitudes, $\left[A\frac{\sin\alpha}{\alpha}\right]$ at the two slits combine to produce interference. The path difference between the rays coming from corresponding points in the slits A_1B_1 and A_2B_2 can be found by drawing a normal from A_1 to A_2R. A_2D is the path difference between the waves from corresponding points of the slits.

In the triangle A_1A_2D $\frac{A_2D}{A_1A_2} = \sin\theta$ or the path difference $A_2D = A_1A_2 \sin\theta = (e + b)\sin\theta$

The corresponding phase difference $(2\beta) = 2\pi(e + b)\sin\theta/\lambda$ ....(2)

Applying the theory of interference on the wave amplitudes $\left[A\frac{\sin\alpha}{\alpha}\right]$ at the two slits gives the resultant wave amplitude (R).

$$R = 2ASin\alpha Cos\beta/\alpha \ldots\ldots\ldots\ldots\ldots.(3)$$

The intensity at P_1 is

$I = R^2 = 4A^2 \sin^2\alpha.\cos^2\beta/\alpha^2$

$= 4I_0 \sin^2\alpha.\cos^2\beta/\alpha^2$ [Since $I_0 = A^2$]...........(4)

Equation (4) represents the intensity distribution on the screen. The intensity at any point on the screen depends on α and β. The intensity of central maximum is $4I_o$. The intensity distribution at different points on the screen can be explained in terms of path difference between the incident and diffracted rays as follows. In equation (4) the term $\cos^2\beta$ corresponds to interference and $\frac{\sin^2\alpha}{\alpha^2}$ corresponds to diffraction. Now, we will shall look at the conditions for interference and diffraction maxima and minima.

Interference maxima and minima: If the path difference $A_2D = (e + b) \sin\theta_n = \pm n\lambda$ where $n = 1, 2, 3\ldots$ then 'θ_n' gives the directions of the maxima due to interference of light waves coming from the two slits. The $\pm$ sign indicates maxima on both sides with respect to the central maximum. On the other hand if the path difference is odd multiples of $\frac{\lambda}{2}$ i.e., $A_2D = (e+b)\sin\theta_n = \pm(2n-1)\frac{\lambda}{2}$, then θ_n gives the directions of minima due to interference of the secondary waves from the two slits on both sides with respect to central maximum.

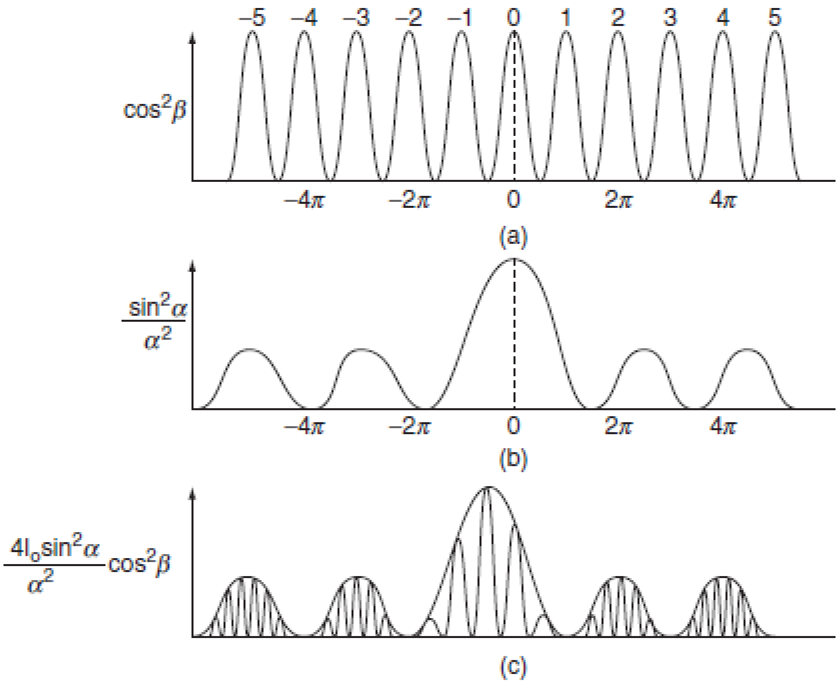
Diffraction maxima and minima: If the path difference $B_1C = e \sin\theta n = \pm n\lambda$, where $n = 1, 2, 3\ldots$ then θ_n gives the directions of diffraction minima. The $\pm$ sign indicates minima on both sides with respect to central maximum. For diffraction maxima $e\sin\theta_n = \pm(2n-1)\frac{\lambda}{2}$ is the condition. The $\pm$ sign indicates maxima on both sides with respect to central maximum.

The intensity distribution on the screen due to double slit diffraction is shown in Fig. 2. Fig. 2(a) represents the graph for interference term, Fig. 2 (b) shows the graph for diffraction term and Fig 2(c) represents the resultant distribution.

Based on the relative values of e and b certain orders of interference maxima are missing in the resultant pattern.

The direction of interference maxima are given as $(e + b) \sin \theta_n = n\lambda$ where $n = 1, 2, 3, \ldots$.. and the directions of diffraction minima are given as $e \sin \theta_m = m\lambda$ where m = 1, 2, 3, …

Figure 2 Intensity distribution due to diffraction at double slit.



For some values of θ_n, the values of e and b are satisfied such that at these positions the interference maxima and the diffraction minima are formed. The combined effect results in missing of certain orders of interference maxima. Now we see certain values of e and b for which interference maxima are missing.

(i) Let $e = b$

Then, $2\, e \sin\theta_n = n\lambda$ and $e \sin\theta_m = m\lambda$

$$\therefore \frac{n}{m} = 2 \quad \text{or} \quad n = 2m$$

If $m = 1, 2, 3 \ldots$ then $n = 2, 4, 6\ldots$ i.e., the interference orders 2, 4, 6 … missed in the diffraction pattern

(ii) If, $2e = b$

Then, $3e \sin \theta_m = n\lambda$ and $e \sin \theta_m = m\lambda$

$$\therefore \quad \frac{n}{m} = 3 \quad \text{or} \quad n = 3m$$

If, $\mu = 1, 2, 3\ldots$ Then, $n = 3, 6, 9\ldots$ i.e the interference orders 3, 6, 9… are missed in the diffraction pattern

(iii) if, $e + b = e$

i.e $b = 0$

the two slits are joined. So, the diffraction pattern is due to a single slit of width $2e$.